

# RAPIDITY GAPS IN QUARK AND GLUON JETS - A PERTURBATIVE APPROACH\*

WOLFGANG OCHS

*Max-Planck-Institut für Physik, (Werner-Heisenberg-Institut)  
Föhringer Ring 6, D-80805 München, Germany*

TOKUZO SHIMADA

*Meiji University, Departement of Physics, Higashi Mita 1-1, Tama  
Kawasaki, Kanagawa 214, Japan*

We derive the probability for rapidity gaps in a parton cascade and investigate the dual connection with hadronic final states. A good description of observations in  $e^+e^-$ -annihilations is obtained by perturbative QCD calculations in MLLA using previously determined parameters (QCD scale  $\Lambda$  and  $k_T$ -cut-off  $Q_0$ ) and applying the parton hadron duality picture. Further predictions are derived; especially, for gaps between jets at variable resolution we predict a strong variation of gap probabilities for small parameters  $y_{cut} \rightarrow 0$  in the transition from jets to hadrons. Large gaps between partons correspond to large spatial separations of colour charges: a colour blanching mechanism by soft processes is suggested.

## 1 Introduction

The occurrence of large rapidity gaps in  $e^+e^-$ -annihilations, or more generally, inside quark and gluon jets, provides an interesting testing ground for models of colour confinement and hadronization.

In case of  $e^+e^-$ -annihilation the primary process is the production of a  $q\bar{q}$  pair. According to a perturbative mechanism<sup>1,2</sup> large rapidity gaps in the hadronic final state occur if in a subsequent process two low mass parton pairs are formed in colour singlet states ( $q\bar{q}$  or  $gg$ ) recoiling against each other. The production rates predicted for these “hard” colour singlets which involve highly virtual intermediate gluons, however, are much smaller than those observed by SLD,<sup>3</sup> by about two orders of magnitude.

A quantitative description of the data is provided by the JETSET MC<sup>4</sup> which combines an initial parton cascade, cut off at a scale  $\sim 1$  GeV, with a string hadronization model. The SLD data are shown in Fig. 1a. The JETSET result fits the data in the full range of rapidity intervals  $\Delta y$ <sup>3</sup>; for large  $\Delta y$  the contribution of  $\tau^+\tau^-$  events becomes important and the dashed histogram represents the purely hadronic contribution.

We will discuss here another approach, originally proposed for the treatment of single inclusive spectra (“Local Parton Hadron Duality”- LPHD<sup>5</sup>) in which the parton cascade is perturbatively evolved further with cutoff

---

\*to appear in Proc. XXIX International Symposium QCD & Multiparticle Production (ISMD99), Brown University, Providence, RI, USA, Aug. 1999, MPI-Pht 99-49

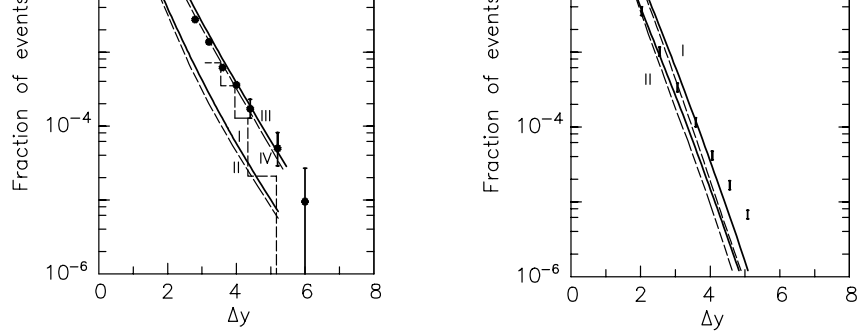


Figure 1. Probability for rapidity gaps of length  $\Delta y$ ; (a) between charged particles in  $e^+e^-$  annihilation: data from SLD<sup>3</sup>, also shown is the JETSET MC for  $q\bar{q}$  initial state without  $\tau^+\tau^-$  events.<sup>3</sup> Furthermore we show our results from ARIADNE MC at the parton level which, in the duality picture, correspond to gaps between all hadrons (curves I, II); an estimate for gaps between charged particles is obtained by multiplying these results with the ratio of gap fractions  $f^{ch}/f^{all}$  from the full MC after string hadronization (curves III, IV) (Parameters used in MC: (I)  $\Lambda = 0.20$  GeV,  $\lambda = 0.015$ ,  $N_f = 3$ ; (II)  $\Lambda = 0.32$  GeV,  $\lambda = 0.015$ ,  $N_f = 5$ ). The data points<sup>3</sup> are moved to the right edge of every interval as appropriate for a cumulative quantity; (b) vertical bars represent the same ARIADNE results (curves I, II in (a)) which are also close to the data; the curves (I) here correspond to our analytical calculation of the Sudakov form factor, the curves (II) include also the cross over correction (full curves:  $\Lambda = 500$  MeV, Durham  $k_T$ ; dashed:  $\Lambda = 350$  MeV, standard  $k_T$ ;  $\lambda = 0.015$  always)

$k_T > Q_0$  at  $Q_0 \gtrsim \Lambda$  with QCD scale  $\Lambda \sim \text{few } 100 \text{ MeV}$ . The results are compared directly to the data without any explicit hadronization phase and the partonic final state is assumed to represent the hadronic one in a dual sense. This simple approach has been applied to a variety of problems, mainly on inclusive quantities, with rather surprising successes.<sup>6,7</sup>

Our present study carries this idea further, in that we consider an observable which tends to become exclusive in the limit of large gaps. Indeed, if colour charges are separated by a large gap, then one might expect, according to conventional wisdom, a neutralization by non-perturbative processes leading to large deviations from the perturbative calculations. At the time of a previous study along these lines<sup>8</sup> the ratio  $Q_0/\Lambda$  was not well determined and only an upper limit of the gap fraction has been given. Meanwhile, from an improved analysis of jet and hadron multiplicities<sup>9</sup> a determination of this ratio has been obtained. Now, the gap rate can be predicted in absolute terms within the perturbatively based duality picture.

## 2 Rapidity Gaps in the Perturbative Parton Cascade

The probability for no radiation into a certain angular interval is given in field theory by the exponential Sudakov form factor<sup>10</sup>, originally derived in QED. In our application we consider the rapidity gap without gluons above the transverse momentum cutoff  $Q_0$ . Let us consider specifically the angular interval between  $\Theta_1$  and  $\Theta_2$  ( $\Theta_1 > \Theta_2$ ), the rapidity is then obtained from  $y = -\ln \tan \frac{\Theta}{2}$ . Let us further denote the probability for emission of a gluon at an angle  $\Theta'$  with the energy  $\omega'$  off a parent parton  $p$  (either a gluon(g) or a quark(q)) as  $\wp_p(\omega', \Theta') = dn_p/d\omega' d\Theta'$ . Then, the Sudakov form factor for

the angular ordered cascade is given by

$$\Delta_p(P, \Theta, Q_0) = \exp(-w_p(P, \Theta, Q_0)) \quad (1)$$

$$w_p(P, \Theta, Q_0) = \int d\omega' \int_{k_\perp > Q_0} d\Theta' \wp_p(\omega', \Theta'), \quad (2)$$

and it represents the probability for no gluon being emitted within the cone of half angle  $\Theta$  at transverse momentum above  $Q_0$  from the parent parton  $p$  of energy  $P = Q/2$ . In particular,

$$\Delta_p(\Theta_2)/\Delta_p(\Theta_1) = \exp(-w_p(\Theta_2) + w_p(\Theta_1)) \quad (3)$$

represents the probability that there is no emission of a gluon with emission angle between  $\Theta_1$  and  $\Theta_2$ . These rates have been calculated in different approximations

*The double logarithmic approximation (DLA)*

The simplest approximation takes into account only the leading contributions from the angle and energy singularities of the gluon emission. The gap probability  $f_p$  is easily calculated analytically.<sup>8</sup> For the symmetrical gap in the *cms* we find a good approximation for not too large gaps  $\Delta y/2 \ll Y$

$$f_p(\Delta y) \simeq \exp(-A_p \Delta y) \quad (4)$$

$$A_p = \frac{4C_p}{b} \ln \frac{Y}{\lambda}, \quad Y = \ln \frac{P\Theta}{Q_0}, \quad \lambda = \ln \frac{Q_0}{\Lambda}, \quad b = \frac{11}{3}N_C - \frac{2}{3}N_f \quad (5)$$

with  $C_g = 3$ ,  $C_q = \frac{4}{3}$ . The gap rate decreases exponentially with  $\Delta y$ . The slope depends sensitively on  $\lambda$  for small  $\lambda$  and  $A \rightarrow \infty$  for  $\lambda \rightarrow 0$ .

*The modified leading logarithmic approximation (MLLA)*

In this improved approximation also the next to leading logarithmic terms are included. Some analytic results have been obtained before.<sup>8</sup> Here we calculate the probability  $w_q$  as in the multiplicity analysis<sup>9</sup>

$$w_q = \int_{Q_0}^{\kappa} d\kappa' \int_{Q_0/\kappa'}^{1-Q_0/\kappa'} dz \frac{\alpha_s(k_T)}{2\pi} \Phi_{qg}(z) \quad (6)$$

by numerical integration, where  $\kappa = Q \sin(\Theta/2)$  is the jet virtuality at opening angle  $\Theta$ , the splitting function is  $\Phi_{qg}(z) = 2C_F(1+(1-z)^2)/z$  and  $k_T$  denotes the transverse momentum; this is taken as  $k_T = z(1-z)\kappa$  (“standard”) or  $k_T = \min(z, 1-z)\kappa$  (“Durham”). The different  $k_T$  lead to somewhat different  $\Lambda$  without changing  $\lambda = 0.015$ .

In this calculation the exponent  $w_q$  is of  $\mathcal{O}(\alpha_s)$ . To next order, processes play a role where a secondary gluon is emitted into the gap although the firstly

emitted gluon is outside the gap, this we call “cross-over effect”. We note that there is no such effect for  $\Theta' < \Theta_2/2$  because of angular ordering, therefore, the maximal effect is a shift by  $\Delta y = \ln 2 \sim 0.7$ . Otherwise, we take the effect into account by a correction factor in DLA accuracy: we multiply (2) under the integral with the probability  $P$  for no secondary emission into the gap, approximately with

$$P = \exp\left(-\frac{1}{2}(w_g(k', \Theta') - w_g(k', \Theta_2 - \Theta'))\right) \quad \text{for} \quad \frac{\Theta_2}{2} < \Theta' < \frac{\Theta_1}{2} \quad (7)$$

$$P = \exp\left(-\frac{1}{2}(w_g(k', \Theta_1 - \Theta') - w_g(k', \Theta_2 - \Theta'))\right) \quad \text{for} \quad \Theta' > \frac{\Theta_1}{2} \quad (8)$$

#### *The Parton Monte Carlo*

As a control of our analytical calculations we compare also with the ARIADNE MC<sup>11</sup> at parton level which is based on similar principles, i.e. cutoff  $k_T > Q_0$  and possibility to choose a small  $\lambda$  parameter. We take parameters (I) determined from a fit to hadron multiplicities<sup>12</sup>; new parameters (II) are determined to improve the fit to jet multiplicities at small  $y_{cut}$ . In the MC we used only  $u$ -quarks with  $m_u = 0$  and kept number of flavours  $N_f$  fixed.

### 3 Comparison with Data

The MC results for the two sets of parameters (I) and (II) are also shown in Fig. 1a. They refer to the gaps between all hadrons in the duality picture. To obtain an estimate for the gaps between charged hadrons only we multiply these curves with the respective ratio derived from the parton and full hadron MC which we parametrized as  $f^{ch}/f^{all} = 1 + 2\Delta y - 0.1(\Delta y)^2$ . After this correction, one observes a very good agreement of this 2-parameter model with the data (at large  $\Delta y$  one should compare to the dashed line).

In Fig. 1b these MC results are represented again as vertical bars for reference, also to the data. The curves (I) represent our analytical calculations based on (6) for two sets of parameters<sup>9</sup>, curves (II) include also the cross over corrections (7),(8). Good agreement of the latter results with the MC is obtained up to  $\Delta y \sim 3 - 4$ , it falls below the MC at higher  $\Delta y$  but could still be close to the experimental data.

### 4 Further Predictions

We note three consequences of our approach which follow directly from the simple DLA formula (4).

*Quark vs. gluon jet*

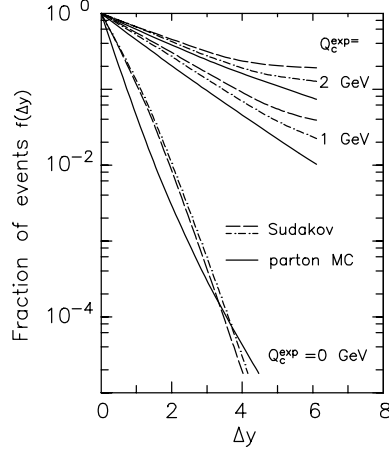


Figure 2. Probability for rapidity gaps of size  $\Delta y$  between jets for different resolution parameters  $y_{cut} = Q_c^{exp}/s$  in the Durham algorithm, so  $Q_c^{exp} = 0$  corresponds to full resolution, i.e. the hadronic final state. The full line represents ARIADNE MC (curve II in Fig. 1), the others the analytical Sudakov calculations without crossover effect (dashed:  $\Lambda = 350$  MeV, standard  $k_T$ ; dash-point:  $\Lambda = 500$  MeV, Durham  $k_T$ ,  $\lambda = 0.015$  always,  $Q = 91$  GeV)

The slope is proportional to  $C_p$  as the Sudakov form factor is derived from the  $\mathcal{O}(\alpha_s)$  gluon emission probability. Then, in DLA, the slope in a gluon jet is larger by  $C_A/C_F = 9/4$  as compared to a quark jet.

#### Energy dependence

The slope behaves like  $A \sim \ln \ln(P/Q_0)$  so the gap distribution gets steeper with increasing jet energy  $P$ .

#### Dependence on cutoff $Q_0$ and jet resolution

In the duality picture, the cutoff  $Q_0$  appears as a hadronization scale which limits the resolution of separate partons. In the evolution equation it can be interpreted also as jet resolution parameter in the Durham algorithm with  $y_{cut} = (Q_{cut}/Q)^2$  with  $Q_0$  replaced by  $Q_{cut}$ . Therefore, we expect a strong dependence of the slope on the jet resolution parameter through  $A \sim -\ln \lambda$ . In the theoretical calculation all hadrons are resolved for  $Q_{cut} \rightarrow Q_0$  in the duality picture, experimentally for  $Q_{cut} \rightarrow 0$ . This mismatch can be resolved by relating<sup>9</sup>  $(Q_{cut}^{th})^2 = (Q_{cut}^{exp})^2 + Q_0^2$ . In Fig. 2 we show predictions for the rapidity gap probability referring now to jets at resolution  $Q_{cut}^{exp} = \sqrt{y_{cut}}Q$  in the Durham algorithm; we neglect the crossover effects which should be small at larger  $Q_{cut}$ . One can see the dramatic rise of  $f(\Delta y)$  at  $\Delta y = 4$  by about

three orders of magnitudes if we replace hadrons by jets at resolution 1 GeV. It will be interesting to verify this new effect.

## 5 Conclusions, a Puzzle and a Physical Picture

We have derived perturbative predictions for rapidity gap distributions using the two parameters  $Q_0$  and  $\Lambda$  from earlier fits to the mean global particle multiplicity. The agreement with the SLD data is quite remarkable and in support of the simple duality picture also in case of this new, partially exclusive, observable. It would be desirable to determine gap fractions from final states with inclusion of neutral particles which would allow a more direct comparison with our calculations. A crucial test of our picture is the strong dependence of the gap distribution on the jet resolution.

Whereas the phenomenological description of the model is successful, the interpretation imposes a serious puzzle: for a large gap, take  $\Delta y = 3$ , the first gluon in each hemisphere is emitted only after a mean lifetime of about 10-20 f, so the perturbative evolution is not disturbed, even if the initial  $q\bar{q}$  pair gets separated far beyond the typical confinement distance.<sup>a</sup>

As the mechanism for global colour blanching is not quantitatively known we may consider phenomenological scenarios. The colour blanching could be mediated by “gluers”<sup>6</sup> at  $k_T \sim Q_0$ , they are expected to cause the production of hadrons with flat rapidity plateau, already in absence of perturbative gluons. In our interpretation there should not be any associated real hadron production in the blanching process as the gap would be refilled. The confinement effects must be weak enough so that the successful perturbative calculation of the gap rate is not invalidated. We consider the possibility that in the field of the separating  $q\bar{q}$  pair (say, inside a tube with virtual partons of  $k_T < Q_0$ ) a very soft new  $\bar{q}q$  pair is produced to ensure confinement at usual distances, and further on at other vertices (Fig. 3). In a more specialized model the picture in Fig. 3 could be realized by effective hadronic vertices<sup>13</sup>. At any rate, it will be interesting to test further the predictions of the perturbative analysis at low energy scales.

## References

1. J.D. Bjorken, S.J. Brodsky and H.J. Lu. *Phys. Lett. B* **286**, 153 (1992).
2. J. Ellis and D.A. Ross, *Z. Phys. C* **70**, 115 (1996).

---

<sup>a</sup>We estimate the lifetime of the virtual quark radiating a gluon with momenta  $k, k_T$  as<sup>6</sup>  $\tau \sim \frac{1}{m_q} \frac{k}{m_q} \sim \frac{k}{k_T^2}$  and take the average of  $\tau$  over  $\omega'$  and  $\Theta'$  as in (2) within the DLA.

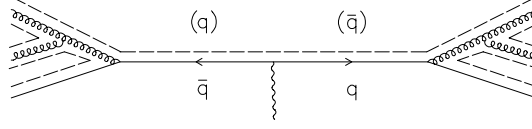


Figure 3. Hadron final state emerging from  $q\bar{q}$  with large rapidity gap: the first gluon emission occurs far outside the confinement region of size  $\sim 1$  f. A possible mechanism is colour blanching by soft  $q\bar{q}$  pairs with  $k_T \lesssim Q_0$  at all vertices.

3. SLD Collaboration: K. Abe *et al*, *Phys. Rev. Lett.* **76**, 115 (1996).
4. T. Sjöstrand, *Comp. Phys. Comm.* **39**, 347 (1986).
5. Ya.I. Azimov, Yu.L. Dokshitzer, V.A. Khoze and S.I. Troyan, *Z. Phys. C* **27**, 65 (1985) and **31**, 213 (1986).
6. Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, “Basics of Perturbative QCD”, Editions Frontière, Gif-sur-Yvette, France (1991).
7. V.A. Khoze and W. Ochs, *Int. J. Mod. Phys. A* **12**, 2949 (1997).
8. W. Ochs and T. Shimada, in *Proc. 33rd Eloisatron Workshop*, Erice, Italy, 19-25 Oct. 1996, ed. L. Cifarelli, A. Kaidalov and V.A. Khoze (World Scientific, Singapore 1998); hep-ph/9706485.
9. S. Lupia and W. Ochs, *Phys. Lett. B* **418**, 214 (1998).
10. V.V. Sudakov, *Sov. Phys. JETP* **3**, 65 (1956).
11. L. Lönnblad, *Comp. Phys. Comm.* **71**, 15 (1992).
12. S. Lupia, W. Ochs, J. Wosiek, *Nucl. Phys. B* **540**, 405 (1999).
13. G. Cohen-Tannoudji and W. Ochs, *Z. Phys. C* **39**, 513 (1988).